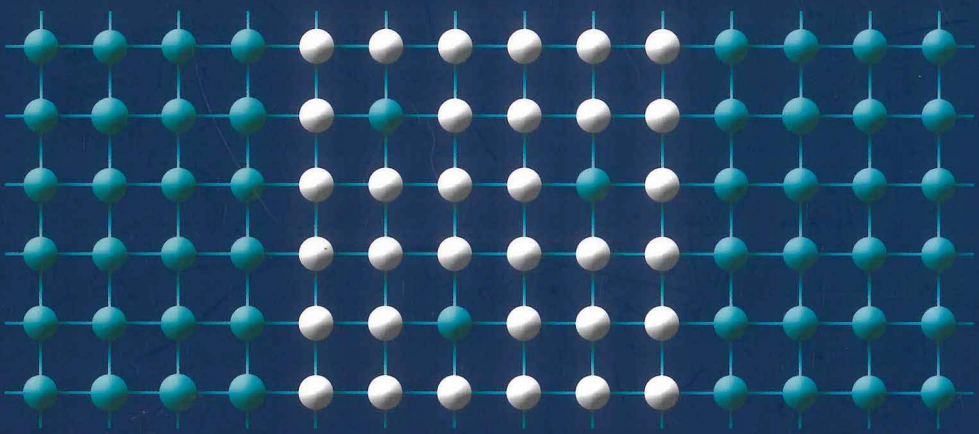
Atomistic Simulation of Quantum Transport in Nanoelectronic Devices



Yu Zhu • Lei Liu

Foreword by
Professor Hong Guo

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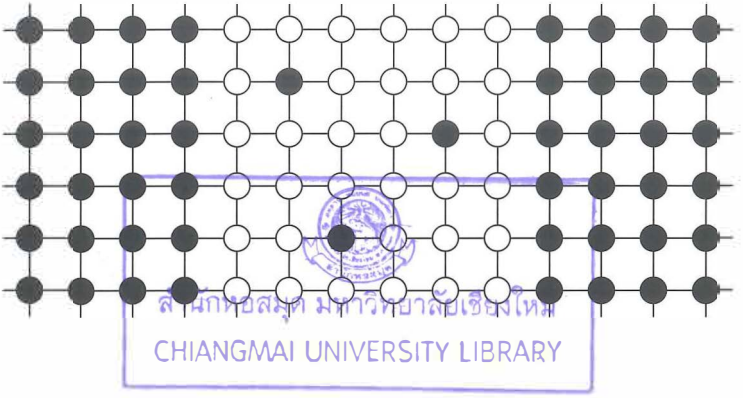

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Contents

Foreword vii

Preface ix

Acknowledgments xiii

1. Introduction 1

   1.1 What is quantum transport? 1
   1.2 Every atom counts 8
   1.3 Disorder and coherent potential 13
   1.4 NECPA-DFT theory and NanoDsim package 18
   1.5 A few words about this monograph 21

2. The NECPA theory 27

   2.1 Two-probe Hamiltonian 27
   2.2 NEGF formalism 30
   2.3 Langreth theorem 32
   2.4 NEGF in steady-state 35
   2.5 Dyson equation 39
   2.6 Current formula 43
   2.7 Surface Green's function 48
   2.8 NECPA equations 49
   2.9 Current conservation and dephasing effect 56
   2.10 A toy model 61

3. The NECPA-LMTO method 67

   3.1 Kohn–Sham equation 67

3.2 Muffin-tin orbital 69
3.3 Structure constant 72
3.4 Tail cancelation 73
3.5 Energy linearization 74
3.6 LMTO Green's function 76
3.7 Screening transform 79
3.8 Physical quantities 81
3.9 Periodicity and Fourier transform 86
3.10 NECPA-LMTO formalism 88
3.11 Self-consistent calculation 91
3.11.1 Flowchart 91
3.11.2 Step-1 calculate structure constant 93
3.11.3 Step-2 calculate self-energy 94
3.11.4 Step-3 make an initial guess 95
3.11.5 Step-4 calculate atomic orbital 96
3.11.6 Step-5 calculate potential parameter 97
3.11.7 Step-6 solve the NECPA equations 98
3.11.8 Step-7 calculate energy moment 100
3.11.9 Step-8 calculate charge density 100
3.11.10 Step-9 calculate charge and dipole 101
3.11.11 Step-10 calculate atomic potential with DFT . . . 102
3.11.12 Step-11 calculate Madelung potential 103
3.11.13 Step-12 calculate total potential 103
3.12 Post-analysis calculation 103
3.12.1 Density of states 103
3.12.2 Transmission coefficient 104
3.12.3 Transmission variation 106
3.12.4 Band structure 107
3.12.5 CPA band structure 107
3.13 Miscellaneous issues 108
3.13.1 Spin degree of freedom 109
3.13.2 Fermi level 109
3.13.3 Linearization center 110
3.13.4 Scalar relativistic equation 111
3.13.5 Wigner–Seitz radius 111

4. NanoDsim: the package design 113

4.1 Do you speak MATLAB? 113
4.2 MATLAB: vectorization technique 118

4.3 MATLAB: hybrid programming 120
4.4 MATLAB: object oriented programming 125
4.5 NanoDsim: overall design 132
4.6 NanoDsim: dsim-solvers 134
4.7 NanoDsim: dsim-calculators 138
4.8 NanoDsim: dsim-classes 139
4.9 NanoDsim: supporting libraries 141
4.10 NanoDsim: implementation and debugging 143

5. NanoDsim: bulk systems 147

5.1 Bulk classes 148
5.1.1 @class_cpaBulk 148
5.1.2 @class_cpaAtom 150
5.1.3 @class_lmtoAtom 151
5.1.4 @class-lmtoOrbital 153
5.2 Bulk solver 154
5.3 Structure constant 156
5.4 Ewald sum technique 159
5.5 Radial equation 165
5.6 Complex contour integral 171
5.7 CPA equations 176
5.8 Fermi level 181
5.9 Bulk calculator: band structure 183
5.10 Bulk calculator: density of states 185

6. NanoDsim: two-probe systems 189

6.1 Two-probe classes 189
6.1.1 @class_necpaTwoProbe 190
6.1.2 @class_necpaAtom 192
6.2 Two-probe solver 193
6.3 Ewald sum technique 195
6.3.1 2d Madelung potential 195
6.3.2 Surface Madelung potential 200
6.3.3 Boundary condition 204
6.4 Surface Green's function 206
6.4.1 Analytically solvable case 206
6.4.2 Recursive method 209
6.4.3 Eigenvalue method 212

6.4.4 A few comments . . . 214
6.5 Real axis integral . . . 216
6.6 k-integral in the Brillouin zone . . . 219
6.6.1 Uniform k-sampling . . . 220
6.6.2 Symmetric k-sampling . . . 221
6.6.3 Time-reversal symmetry . . . 226
6.7 NECPA equations . . . 232
6.8 Fermi level alignment . . . 237
6.9 Two-probe calculator: transmission coefficient . . . 239
6.10 Verification of the implementation . . . 241

7. NanoDsim: optimization and parallelization 247
7.1 Performance analysis . . . 247
7.2 Memory issues . . . 250
7.3 Speed issues . . . 253
7.3.1 Order-N methods . . . 253
7.3.2 Iterative methods . . . 254
7.3.3 Direct methods . . . 257
7.3.4 Summary . . . 260
7.4 Principal layer algorithm . . . 261
7.4.1 Retarded Green's function . . . 261
7.4.2 Lesser Green's function . . . 265
7.4.3 Transmission coefficient . . . 266
7.4.4 Cost estimate . . . 267
7.4.5 Implementation details . . . 269
7.5 MATLAB interface to MPI . . . 269
7.6 Parallelization . . . 272
7.7 Benchmark . . . 274
7.8 Convergence issues . . . 276
7.9 Error analysis . . . 278

8. Kaleidoscope of the physics in disordered systems 283
8.1 Simple examples: bulk Cu, Fe, Co, Ni . . . 283
8.2 CPA vs supercell: Cu/Co alloy . . . 286
8.3 Si with uniaxial strain . . . 288
8.4 Band offset of GaAs/$Al_xGa_{1-x}As$ heterojunctions . . . 292
8.5 NECPA vs supercell: Cu/Co interface . . . 294
8.6 Si transistors with localized doping . . . 298

8.7 Graphene transistors with disorder scattering 302
8.8 Fe/MgO/Fe tunnel junctions 307
8.9 Cu films with surface scattering 311
8.10 Concluding remarks 315

Appendix 319

A.1 Atomic units 319
A.2 Phase diagram of the toy model 320
A.3 Classical transport vs quantum transport 324
A.3.1 Drift-Diffusion model 325
A.3.2 Effective-Mass model 327
A.3.3 Numerical results 333
A.4 Lehmann spectrum of NEGF 334
A.5 Low concentration approximation 339
A.5.1 Multiple scattering theory 339
A.5.2 Transmission coefficient and transmission variation 342
A.6 Scattering states approach 345
A.6.1 Bulk states 345
A.6.2 Wave scattering 347
A.6.3 Transmission coefficient 350
A.6.4 Further discussion: group velocity 352
A.6.5 Further discussion: number of modes 352
A.6.6 Further discussion: numerical issues 353
A.6.7 Summary 355
A.7 Density matrix in clean bulk systems 355
A.8 Connection to the CPA-NVC theory 357
A.9 Explicit expressions of XC-functionals 358
A.9.1 LDA: Perdew and Zunger (1981) 359
A.9.2 GGA: Perdew, Burke, and Ernzerhof (1996) . . . 360
A.9.3 MBJ: Tran and Blaha (2009) 361
A.9.4 A few comments 363
A.10 Complex-valued and real-valued spherical harmonics . . . 363
A.11 Gaunt coefficients 365
A.12 Eigensolutions of TST and TSC matrices 366
A.13 Proof of the Wronskian identity 368
A.14 Numerical proof 369
A.15 Transmission coefficient in the LMTO method 370
A.16 Specular scattering vs diffusive scattering 373
A.17 Fill the space with atomic spheres 376

A.17.1 Regular structures 376
A.17.2 Irregular structures 378
A.18 Symmetric k-sampling 380
A.19 Unfolding algorithm 385
A.20 Mixing algorithms 386
A.21 Modified Fermi pole summation technique 389
A.22 Field effect transistor with gate terminals 391
A.23 Algorithms for solving the Poisson equation 393
A.23.1 Numerical discretization 393
A.23.2 Algorithms in 1d case 395
A.23.3 Algorithms in 2d and 3d cases 398
A.23.4 Nonorthogonal Poisson box 399
A.23.5 Nonlinear Poisson equation 400
A.24 Locality in nonequilibrium 402
A.25 Lanczos algorithm 403
A.26 Preconditioner designed for quantum transport 407
A.27 Content of the affiliated CD 411
A.27.1 NanoDsim package 411
A.27.2 ResearchCode package 411