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An Invitation to  
the Theory of  
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# An Invitation to the Theory of the Hybridizable Discontinuous Galerkin Method

Projections, Estimates, Tools



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