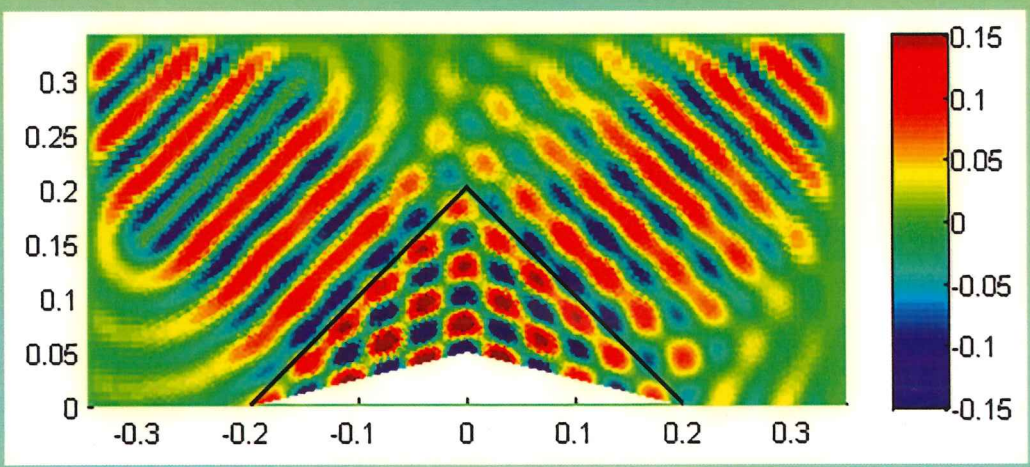
TEXTBOOKS IN MATHEMATICS

COMPUTATIONAL PARTIAL DIFFERENTIAL EQUATIONS USING MATLAB®

SECOND EDITION



Jichun Li
Yi-Tung Chen

CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK

สำนักหอสมุด มหาวิทยาลัยเชียงใหม่
012764.38
i2684840
b1670437x

Computational Partial Differential Equations Using MATLAB®

Second Edition

Jichun Li
Yi-Tung Chen

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an informa business
A CHAPMAN & HALL BOOK

Contents

Preface xi

Acknowledgments xiii

1 Brief Overview of Partial Differential Equations 1
- 1.1 The parabolic equations . . . 1
- 1.2 The wave equations . . . 2
- 1.3 The elliptic equations . . . 3
- 1.4 Differential equations in broader areas . . . 3
  - 1.4.1 Electromagnetics . . . 3
  - 1.4.2 Fluid mechanics . . . 4
  - 1.4.3 Groundwater contamination . . . 5
  - 1.4.4 Petroleum reservoir simulation . . . 6
  - 1.4.5 Finance modeling . . . 7
  - 1.4.6 Image processing . . . 8
- 1.5 A quick review of numerical methods for PDEs . . . 9

References . . . 11

2 Finite Difference Methods for Parabolic Equations 13
- 2.1 Introduction . . . 13
- 2.2 Theoretical issues: stability, consistence, and convergence . . . 16
- 2.3 1-D parabolic equations . . . 18
  - 2.3.1 The θ-method and its analysis . . . 18
  - 2.3.2 Some extensions . . . 23
- 2.4 2-D and 3-D parabolic equations . . . 27
  - 2.4.1 Standard explicit and implicit methods . . . 27
  - 2.4.2 The ADI methods for 2-D problems . . . 30
  - 2.4.3 The ADI methods for 3-D problems . . . 32
- 2.5 Numerical examples with MATLAB codes . . . 35
- 2.6 Bibliographical remarks . . . 37
- 2.7 Exercises . . . 38

References . . . 41

3 Finite Difference Methods for Hyperbolic Equations 43
- 3.1 Introduction . . . 43
- 3.2 Some basic difference schemes . . . 44
- 3.3 Dissipation and dispersion errors . . . 47

3.4 Extensions to conservation laws . . . 49
3.5 The second-order hyperbolic equations . . . 50
3.5.1 The 1-D case . . . 50
3.5.2 The 2-D case . . . 53
3.6 Numerical examples with MATLAB codes . . . 57
3.7 Bibliographical remarks . . . 59
3.8 Exercises . . . 60
References . . . 62

4 Finite Difference Methods for Elliptic Equations 65
4.1 Introduction . . . 65
4.2 Numerical solution of linear systems . . . 67
4.2.1 Direct methods . . . 67
4.2.2 Simple iterative methods . . . 69
4.2.3 Modern iterative methods . . . 72
4.3 Error analysis with a maximum principle . . . 74
4.4 Some extensions . . . 76
4.4.1 Mixed boundary conditions . . . 77
4.4.2 Self-adjoint problems . . . 78
4.4.3 A fourth-order scheme . . . 78
4.5 Numerical examples with MATLAB codes . . . 81
4.6 Bibliographical remarks . . . 84
4.7 Exercises . . . 84
References . . . 86

5 High-Order Compact Difference Methods 89
5.1 One-dimensional problems . . . 89
5.1.1 Spatial discretization . . . 89
5.1.2 Dispersive error analysis . . . 94
5.1.3 Temporal discretization . . . 98
5.1.4 Low-pass spatial filter . . . 103
5.1.5 Numerical examples with MATLAB codes . . . 104
5.2 High-dimensional problems . . . 121
5.2.1 Temporal discretization for 2-D problems . . . 121
5.2.2 Stability analysis . . . 123
5.2.3 Extensions to 3-D compact ADI schemes . . . 124
5.2.4 Numerical examples with MATLAB codes . . . 125
5.3 Other high-order compact schemes . . . 133
5.3.1 One-dimensional problems . . . 133
5.3.2 Two-dimensional problems . . . 135
5.4 Bibliographical remarks . . . 138
5.5 Exercises . . . 138
References . . . 141

6 Finite Element Methods: Basic Theory 145
6.1 Introduction to one-dimensional problems 145
6.1.1 The second-order equation 145
6.1.2 The fourth-order equation 148
6.2 Introduction to two-dimensional problems 152
6.2.1 The Poisson equation 152
6.2.2 The biharmonic problem 154
6.3 Abstract finite element theory 155
6.3.1 Existence and uniqueness 156
6.3.2 Stability and convergence 157
6.4 Examples of conforming finite element spaces 158
6.4.1 Triangular finite elements 159
6.4.2 Rectangular finite elements 163
6.5 Examples of nonconforming finite elements 164
6.5.1 Nonconforming triangular elements 164
6.5.2 Nonconforming rectangular elements 165
6.6 Finite element interpolation theory 167
6.6.1 Sobolev spaces 167
6.6.2 Interpolation theory 169
6.7 Finite element analysis of elliptic problems 173
6.7.1 Analysis of conforming finite elements 173
6.7.2 Analysis of nonconforming finite elements 175
6.8 Finite element analysis of time-dependent problems 177
6.8.1 Introduction 177
6.8.2 FEM for parabolic equations 178
6.9 Bibliographical remarks 185
6.10 Exercises 186
References 188

7 Finite Element Methods: Programming 193
7.1 FEM mesh generation 193
7.2 Forming FEM equations 198
7.3 Calculation of element matrices 199
7.4 Assembly and implementation of boundary conditions 204
7.5 The MATLAB code for P_1 element 205
7.6 The MATLAB code for the Q_1 element 208
7.7 Bibliographical remarks 213
7.8 Exercises 214
References 217

8 Mixed Finite Element Methods 219
8.1 An abstract formulation 219
8.2 Mixed methods for elliptic problems 223
8.2.1 The mixed variational formulation 223
8.2.2 The mixed finite element spaces 225
8.2.3 The error estimates 229

8.3 Mixed methods for the Stokes problem . . . 232
8.3.1 The mixed variational formulation . . . 232
8.3.2 Mixed finite element spaces . . . 234
8.4 An example MATLAB code for the Stokes problem . . . 238
8.5 Mixed methods for viscous incompressible flows . . . 252
8.5.1 The steady Navier-Stokes problem . . . 252
8.5.2 The unsteady Navier-Stokes problem . . . 254
8.6 Bibliographical remarks . . . 255
8.7 Exercises . . . 256
References . . . 259

9 Finite Element Methods for Electromagnetics 261
9.1 Introduction to Maxwell's equations . . . 261
9.2 The time-domain finite difference method . . . 263
9.2.1 The semi-discrete scheme . . . 263
9.2.2 The fully discrete scheme . . . 272
9.3 The time-domain finite element method . . . 285
9.3.1 The mixed method . . . 285
9.3.2 The standard Galerkin method . . . 290
9.3.3 The discontinuous Galerkin method . . . 293
9.4 The frequency-domain finite element method . . . 298
9.4.1 The standard Galerkin method . . . 298
9.4.2 The discontinuous Galerkin method . . . 299
9.4.3 The mixed DG method . . . 303
9.5 Maxwell's equations in dispersive media . . . 305
9.5.1 Isotropic cold plasma . . . 306
9.5.2 Debye medium . . . 310
9.5.3 Lorentz medium . . . 313
9.5.4 Double-negative metamaterials . . . 315
9.6 Bibliographical remarks . . . 323
9.7 Exercises . . . 324
References . . . 325

10 Meshless Methods with Radial Basis Functions 331
10.1 Introduction . . . 331
10.2 The radial basis functions . . . 332
10.3 The MFS-DRM . . . 335
10.3.1 The fundamental solution of PDEs . . . 335
10.3.2 The MFS for Laplace's equation . . . 338
10.3.3 The MFS-DRM for elliptic equations . . . 341
10.3.4 Computing particular solutions using RBFs . . . 344
10.3.5 The RBF-MFS . . . 346
10.3.6 The MFS-DRM for the parabolic equations . . . 346
10.4 Kansa's method . . . 348
10.4.1 Kansa's method for elliptic problems . . . 348

10.4.2 Kansa's method for parabolic equations 349
10.4.3 The Hermite-Birkhoff collocation method 350
10.5 Numerical examples with MATLAB codes 352
10.5.1 Elliptic problems 352
10.5.2 Biharmonic problems 359
10.6 Coupling RBF meshless methods with DDM 366
10.6.1 Overlapping DDM 367
10.6.2 Non-overlapping DDM 368
10.6.3 One numerical example 369
10.7 Bibliographical remarks 372
10.8 Exercises 372
References 373

11 Other Meshless Methods **379**
11.1 Construction of meshless shape functions 379
11.1.1 The smooth particle hydrodynamics method 379
11.1.2 The moving least-square approximation 381
11.1.3 The partition of unity method 382
11.2 The element-free Galerkin method 384
11.3 The meshless local Petrov-Galerkin method 386
11.4 Bibliographical remarks 389
11.5 Exercises 389
References 390

Appendix A Answers to Selected Problems **393**

Index **405**