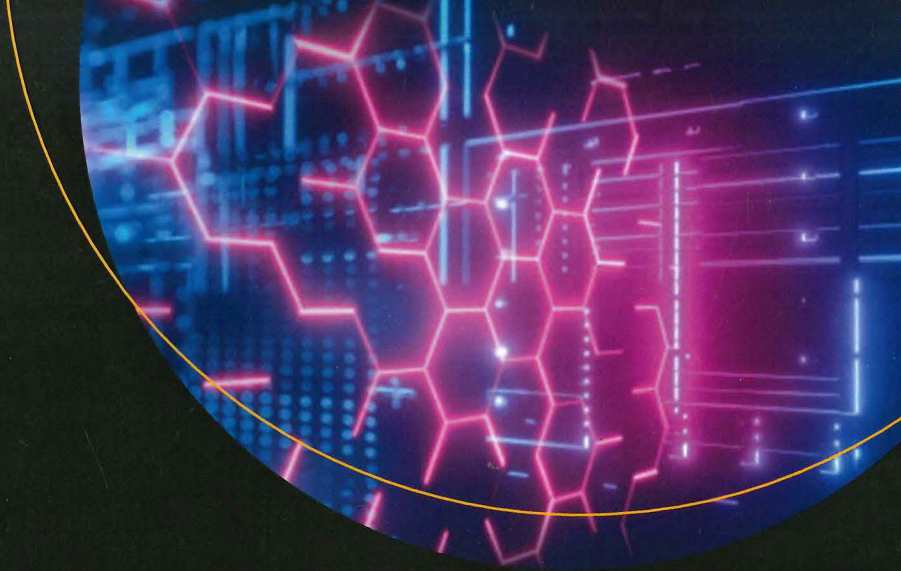




# Quantum Machine Learning with Python

Using Cirq from Google Research  
and IBM Qiskit

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Santanu Pattanayak

Apress®

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and IBM Qiskit



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