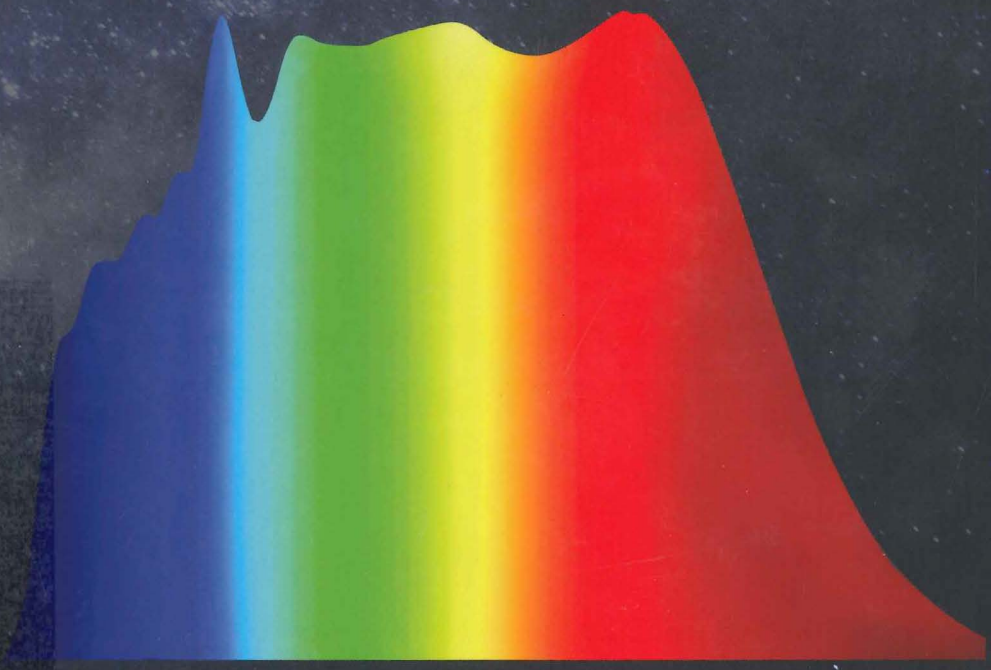
Lectures on Quantum Mechanics and Attractors

Alexander Komech



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Lectures on Quantum Mechanics and Attractors

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