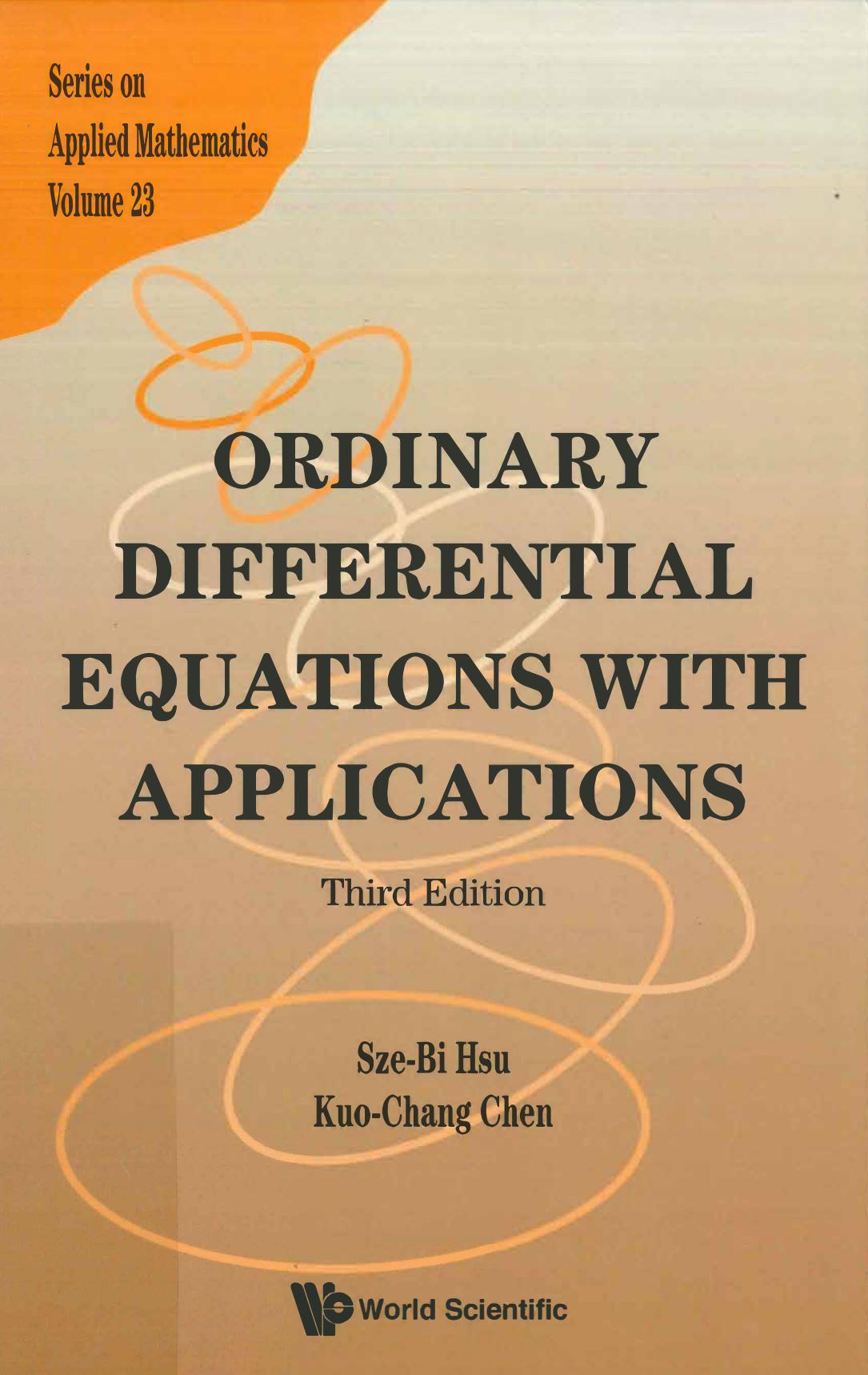


Series on  
Applied Mathematics  
Volume 23



# ORDINARY DIFFERENTIAL EQUATIONS WITH APPLICATIONS

Third Edition

Sze-Bi Hsu  
Kuo-Chang Chen



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Sze-Bi Hsu  
Kuo-Chang Chen

*National Tsing Hua University, Taiwan*



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