Series on
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Volume 23

ORDINARY DIFFERENTIAL EQUATIONS WITH APPLICATIONS

Third Edition

Sze-Bi Hsu
Kuo-Chang Chen

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Sze-Bi Hsu
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