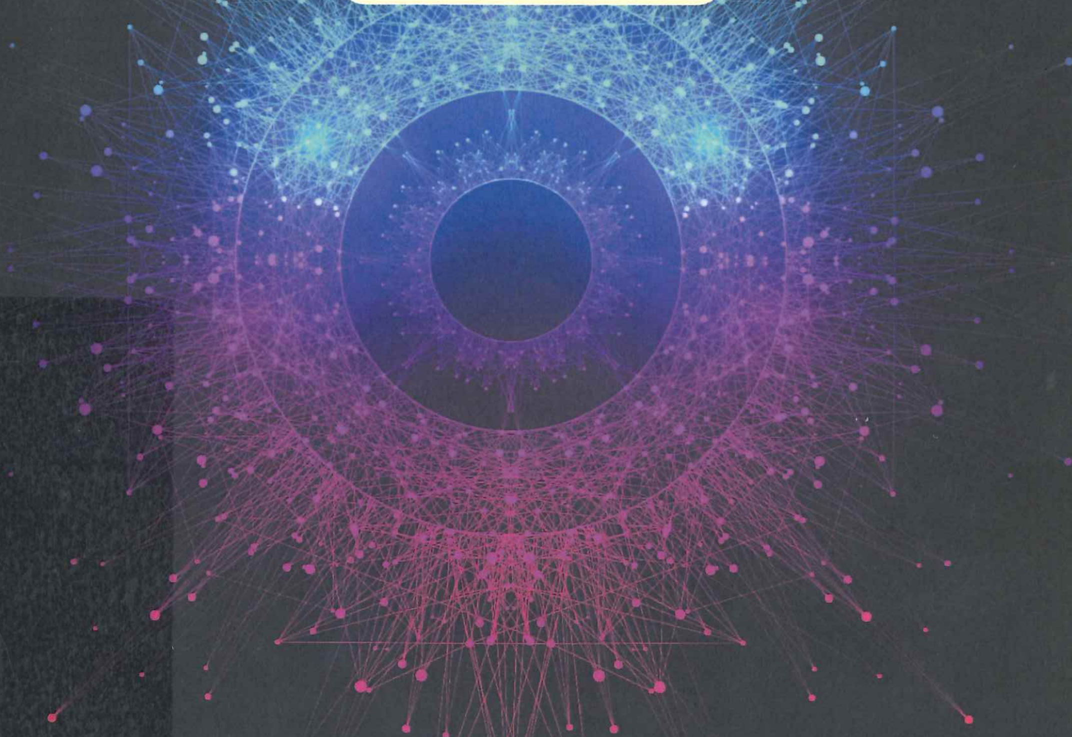
Lecture Notes Series on Computing - Vol. 15

M. H. Alsuwaiyel

Algorithms

Design Techniques and Analysis

Second Edition



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Algorithms
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