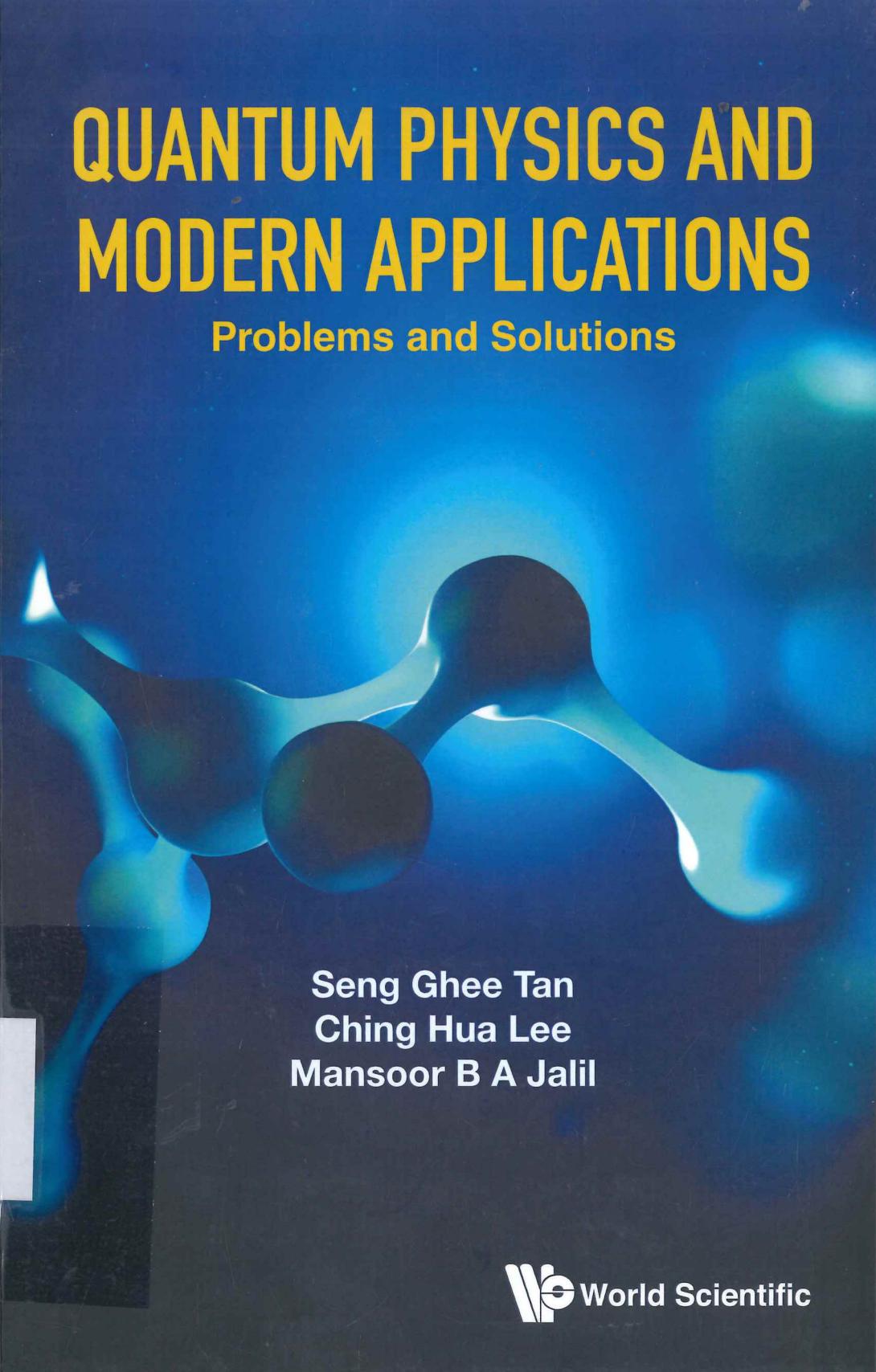


QUANTUM PHYSICS AND MODERN APPLICATIONS

Problems and Solutions



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**Seng Ghee Tan
Ching Hua Lee
Mansoor B A Jalil**



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QUANTUM PHYSICS AND MODERN APPLICATIONS

Problems and Solutions



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