



Numerical Analysis and Scientific Computing Series

# AN INTRODUCTION TO NUMERICAL METHODS

A MATLAB® Approach

FIFTH EDITION

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Abdelwahab Kharab  
Ronald B. Guenther



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A CHAPMAN & HALL BOOK

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# An Introduction to Numerical Methods

A MATLAB® Approach

FIFTH EDITION



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