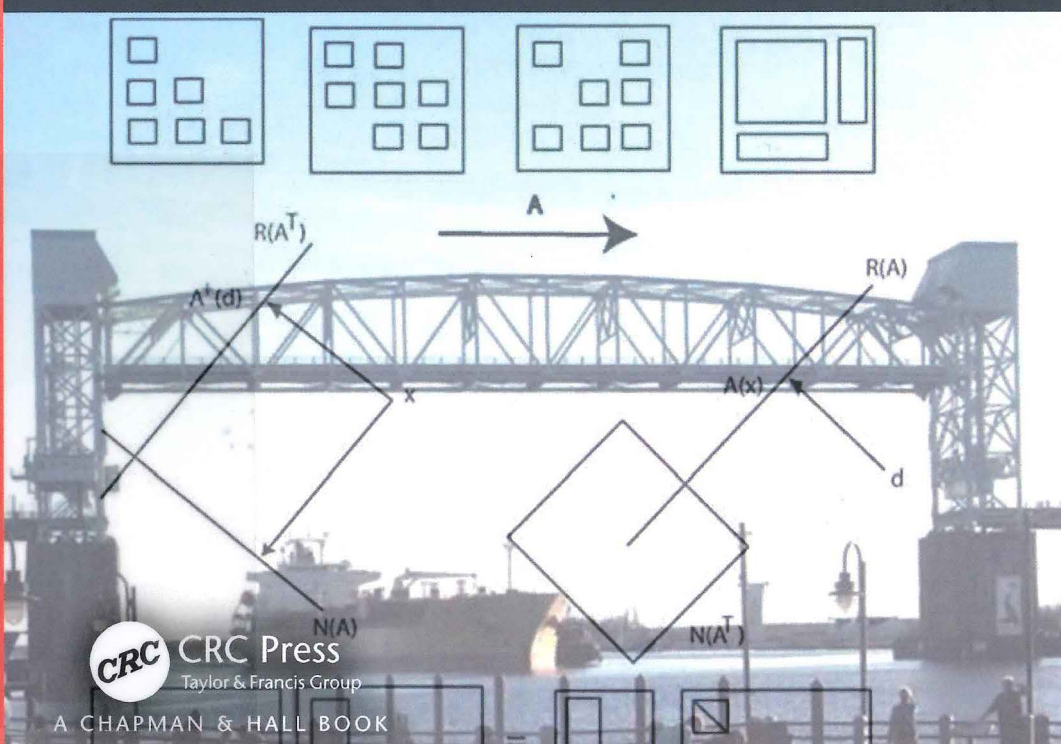
TEXTBOOKS IN MATHEMATICS

COMPUTATIONAL LINEAR ALGEBRA

with Applications and MATLAB® Computations

Robert E. White



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Computational Linear Algebra

with Applications and MATLAB® Computations

Robert E. White

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