TEXTBOOKS IN MATHEMATICS

A FIRST COURSE IN FUZZY LOGIC

FOURTH EDITION

Hung T. Nguyen
Carol L. Walker
Elbert A. Walker

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