



Sheldon M. Ross

Thirteenth Edition

Introduction to
**Probability
Models**



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Introduction to Probability Models

สำนักหอสมุด มหาวิทยาลัยเชียงใหม่

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Thirteenth Edition

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สำนักหอสมุด มหาวิทยาลัยเชียงใหม่
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